

DEPARTMENT of MATHEMATICS

MAT302-120

Instructor:

CLT:

DEFINITE INTEGRAL

OBJECTIVE: To calculate the value of the definite integral and plot the graph of a given function.

a) The integral of $f(x) = \cos x^2$ can be found by Second Fundamental Theorem of Calculus because it is continuous. The antiderivative of $f(x)$ on $[-\pi, 0]$ can be given by $g(x) = \int_{-\pi}^x \cos(t^2) dt$. $g(x)' = f(x)$ can be verified as shown below.

> *with (plots) :*

> $f := x \rightarrow \cos(x^2);$

$$f := x \mapsto \cos(x^2) \quad (1)$$

> $g := x \rightarrow \text{int}(\cos(t^2), t = -\pi..x);$

$$g := x \mapsto \int_{-\pi}^x \cos(t^2) dt \quad (2)$$

> $\text{diff}(g(x), x);$

$$\cos(x^2) \quad (3)$$

b) similarly, $h(x)$ can be defined as $h(x) = \int_{-1}^x \cos(t^2) dt$ verify that $h(x)' = f(x)$ as follows.

> $h := x \rightarrow \text{int}(\cos(t^2), t = -1..x);$

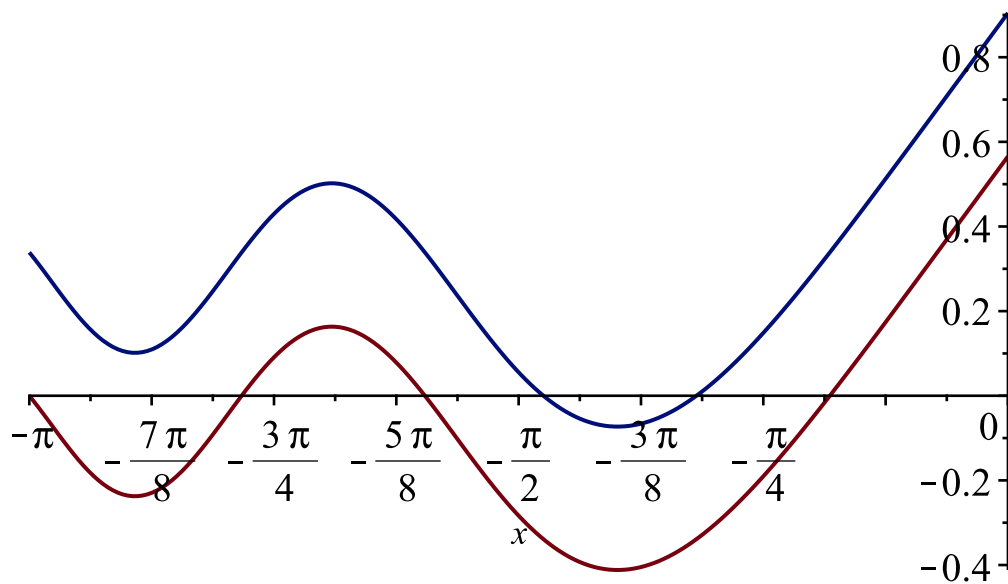
$$h := x \mapsto \int_{-1}^x \cos(t^2) dt \quad (4)$$

> $\text{diff}(h(x), x);$

$$\cos(x^2) \quad (5)$$

c) The graph of $g(x)$ and $h(x)$ can be plotted as follows;

> $\text{plot}([g(x), h(x)], x = -\pi..0);$



Graph output analysis: these functions differ by a constant approximately equal to -0.3. To verify this results we

notice that the difference $g(x) - h(x)$ can be given as

$$\int_{-\pi}^x \cos(t^2) dt - \int_{-1}^x \cos(t^2) dt = \int_{-\pi}^x \cos(t^2) dt + \int_x^{-1} \cos(t^2) dt = \int_{-\pi}^{-1} \cos(t^2) dt$$

> $\text{Int}(\cos(t^2), t = -\pi \dots -1);$

$$\int_{-\pi}^{-1} \cos(t^2) dt$$

(6)

> $\text{evalf}(\%);$

$$-0.3388307243$$

(7)

with the accuracy of five decimal digits the constant is equal to -0.33883